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A method for the clipped intensity correlation measurement†

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Abstract. We propose an alternative scheme for the clipped light intensity correlation measurement which is complementary to the usual scheme as proposed by Jakeman and Pike. The new scheme gives essentially the same statistical accuracy for the linewidth measurement and furthermore, it has some practical advantages when the mean number of counts per sampling time is sufficiently large.

The essence of the digital intensity correlation technique consists of measuring the autocorrelation function of the photocount $n(t, T)$, which is defined as the number of photocounts detected during an interval $(t, t+T)$. Under a condition that the sampling time T is much smaller than the correlation time of the intensity fluctuations τ_c, one can easily show that (Mehta 1970)

$$\langle n(t_1, T)n(t_2, T)\rangle = (\alpha T)^2\langle I(t_1)I(t_2)\rangle \qquad t_2 > t_1 \tag{1}$$

where α is the quantum efficiency of the photodetector and $I(t)$ the instantaneous intensity of the light incident on the point photocathode. For the so called 'Gaussian–Lorentzian' light of bandwidth $\Gamma\,(=1/\tau_c)$ one has (assuming the spatial coherence factor is made close to unity) (Mandel and Wolf 1965)

$$\langle I(t_1)I(t_2)\rangle = \langle I\rangle^2\{1+\exp(-2\Gamma|t_1-t_2|)\} \tag{2}$$

or

$$\langle n(t_1, T)n(t_2, T)\rangle = (\alpha TI)^2\{1+\exp(-2\Gamma|t_1-t_2|)\} = \langle n\rangle^2\{1+\exp(-2\Gamma|t_1-t_2|)\}. \tag{3}$$

Regarding $n(t, T)$ as a random process one can introduce a joint photocount distribution function $P(n_1, n_2)$ = probability of detecting n_1 counts at (t_1, t_1+T) followed by detection of n_2 counts at (t_2, t_2+T). The photocount autocorrelation function (1) is then given by

$$\langle n(t_1, T)n(t_2, T)\rangle = \sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty} n_1 n_2 P(n_1, n_2). \tag{4}$$

In practice computation of the photocount autocorrelation function requires a complex

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digital electronics and is rather time consuming. To overcome this difficulty one normally uses the so called single channel clipping technique first introduced by Jakeman and Pike (1969). The principle is as follows.

If we define a binary random variable $n_k(0)$ by

$$n_k(0) = 1 \qquad \text{if } n(0, T) > k$$
$$n_k(0) = 0 \qquad \text{if } n(0, T) \leqslant k \tag{5}$$

where k is an integer or zero, then one has for the clipped autocorrelation function

$\langle n_k(0)n(t, T)\rangle$

$$= \sum_{n_1=k+1}^{\infty} \sum_{n_2=0}^{\infty} n_2 P(n_1 n_2) = \sum_{n_2=0}^{\infty} \left(\sum_{n_1=0}^{\infty} n_2 P(n_1, n_2) - \sum_{n_1=0}^{k} n_2 P(n_1, n_2) \right)$$
$$= \sum_{n_2=0}^{\infty} n_2 P(n_2) - \sum_{n_1=0}^{k} \sum_{n_2=0}^{\infty} n_2 P(n_1, n_2) = \langle n\rangle - \langle n_k(0)n(t, T)\rangle_C \tag{6}$$

where we have introduced the complementary clipped autocorrelation function by

$$\langle n_k(0)n(t, T)\rangle_C \equiv \sum_{n_1=0}^{k} \sum_{n_2=0}^{\infty} n_2 P(n_1, n_2). \tag{7}$$

Jakeman and Pike (1969) have shown that for the 'Gaussian–Lorentzian' light the clipped autocorrelation function is

$$\langle n_k(0)n(t, T)\rangle = \langle n\rangle \langle n_k\rangle + \langle n\rangle \langle n_k\rangle \frac{1+k}{1+\langle n\rangle} e^{-2\Gamma t} \tag{8}$$

and where $\langle n_k\rangle$ is given, for $T \ll \tau_C$, by

$$\langle n_k\rangle = \left(\frac{\langle n\rangle}{1+\langle n\rangle} \right)^{k+1}. \tag{9}$$

The complementary clipped autocorrelation function in this case is thus

$$\langle n_k(0)n(t, T)\rangle_C = \langle n\rangle (1 - \langle n_k\rangle) - \langle n\rangle \langle n_k\rangle \frac{1+k}{1+\langle n\rangle} e^{-2\Gamma t}. \tag{10}$$

These results show that both clipping techniques can be used to measure linewidth Γ. Figure 1 shows the two quantities $\langle n_k(0)n(t, T)\rangle$ and $\langle n_k(0)n(t, T)\rangle_C$ as a function of time for $k = 0$ and $\langle n\rangle = 5$. An experimental arrangement of a singly-clipped autocorrelator has been described by Foord *et al* (1970). With this type of arrangement one can easily perform the measurement of both the clipped and the complementary clipped correlation functions.

One possible implementation of this type of clipping gate is illustrated in figure 2. In the circuit the one-bit sampling registers G1 and G2 are being enabled and reset alternatively by the clock pulses. This way one can avoid the dead time associated with the resetting. If one uses the high-speed TTL integrated circuits the sampling time can be as short as 50 ns. The outputs from G3 or G4 go to the input of the shift register which functions as the digital delay line in the correlator. If one wishes to have higher k values, one simply cascades more flip–flops.

As far as operation of the correlator is concerned the only difference between the normal and the complementary clipping is as follows. For each sampling the former

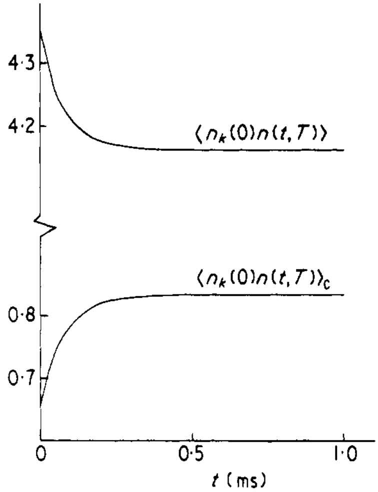

Figure 1. Clipped and complementary clipped photocount autocorrelation functions for the case $k = 0$, $\langle n \rangle = 5$ and $\tau_C = 150$ μs.

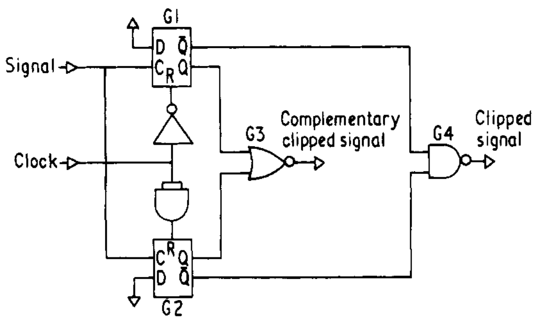

Figure 2. $k = 0$ clipping gate used in our correlator.

produce a binary number n_k while the latter $(1-n_k)$. Thus at time $t_l = lT$ later, the channel number l receives counts of $n_k n(lT)$ in the former and $(1-n_k)n(lT)$ in the latter. After N sampling then, the content of the lth channel is (suppressing the argument T in n)

$$\hat{S}(t_l) = \sum_{r=1}^{N} n_k(t_r)n(t_{r+l}) \qquad \text{normal clipping}$$

$$\hat{S}_c(t_l) = \sum_{r=1}^{N} (1-n_k(t_r))n(t_{r+l}) \qquad \text{complementary clipping}$$

and

$$\hat{S}_c(t_l) = \hat{n} - \hat{S}(t_l)$$

where $\hat{n} = \Sigma_{r=1}^{N} n(t_r)$, is the total counts input to the correlator during N samplings and is a number usually recorded in the correlator. We see thus the information content of $\hat{S}(t_l)$ and $\hat{S}_c(t_l)$ is identical and one can get the former from the latter. The statistical accuracy of the two methods therefore, cannot, in principle, be any different.

There is, however, one practical advantage in the complementary scheme when $\langle n\rangle$ is large. Take for instance the $k = 0$ case. Referring back to expressions (8), (9) and (10), we notice that on the average, the maximum counts obtained per sampling are significantly different for the two methods:

$$2\langle n\rangle\langle n_0\rangle = 2\langle n\rangle\frac{\langle n\rangle}{1+\langle n\rangle} \qquad \text{normal clipping}$$

$$\langle n\rangle(1+\langle n\rangle) = \frac{\langle n\rangle}{1+\langle n\rangle} \qquad \text{complementary clipping.}$$

While the former approaches $2\langle n\rangle$ for large $\langle n\rangle$, the latter is always less than 1. This is significant for a correlator employing a buffer storage of finite bits, because one can minimize the overflow of data at high counting rate and sampling time. For example, the correlator designed by one of us (see Chen and Lai 1972) employs a 4-bits buffer storage counter after the coincident gate at each channel and the consequent divided output (divided by 16) is read into the memory of a multichannel analyser every 1·3 ms. (There are 130 channels in the correlator and the serial transfer rate of data to MCA is 10 μs per bit.) Thus one has to make sure that the coincident output (per channel) is less than 16 during the 1·30 ms period. Figure 3 shows an example of this point. The scattered light is from Brownian particles of diameter 0·481 μm suspended in methanol.

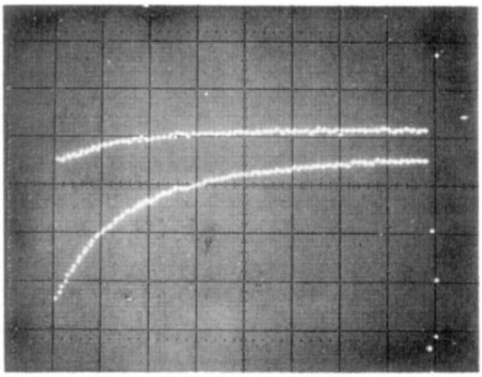

Figure 3. $\langle n_0(0)n(t)\rangle$, upper curve and $\langle n_0(0)n(t)\rangle_C$, lower curve, for the same light scattered from suspended Brownian particles in methanol.

The counting rate is $R = 3\times10^4$ counts s^{-1} and the sampling time $T = 200$ μs per channel. Thus $\langle n\rangle = 6$ and in 1·30 ms there are about 6 samplings. If one uses the normal clipping one needs a buffer storage of more than $2\langle n\rangle\times 6 = 72$ while if one uses the complementary clipping one needs only the buffer storage of about 6. We see clearly that the upper curve, corresponding to the output of the normal clipping, suffers from overflow problems and the exponential decay is wiped out. The lower curve represents the result of the complementary clipping and there is no problem of overflow. This problem becomes rather acute when one performs heterodyne measurement where one has to mix-in a strong local oscillator beam. For a correlator which directly couples outputs of the coincident gates to a multibits semiconductor memory the above mentioned advantages are greatly reduced. However, the basic reason for building a buffered storage correlator is in its simplicity and economy. The limitation of the latter type of correlator at high counting rate is discussed in detail by Chen and Lai (1972).

Another application of the complementary clipping is in measuring $p(0, 0) \equiv \langle n_0(t_1)n_0(t_2)\rangle_C$ by two channel clipping. When one performs a small angle scattering experiment where dust particles in the sample are always a problem, measurement of $p(0, 0)$ effectively avoids this problem, by essentially shutting off the correlator every time the dust particles get into the beam.

In conclusion we point out that both clipping schemes are useful in a correlator and while the normal clipping is the natural one to use for $\langle n\rangle < 1$, the complementary clipping is useful for the $\langle n\rangle \gg 1$ case.

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